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Qualitative and Numerical Analysis of a Cosmological Model Based on a Phantom Scalar Field with Self-Interaction

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Abstract—We investigate the asymptotic behavior of a cosmological model based on a phantom scalar field by qualitative analysis of the set of differential equations. We show that, as opposed to models with a classical scalar field, such models have stable asymptotic solutions with a constant value of the potential both in the infinite past and the infinite future. We also develop numerical models of the cosmological evolution with a phantom scalar field.

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1. INTRODUCTION

Previously, a cosmological model was created, based on statistical systems of scalar charged particles interacting via a phantom scalar field possessing negative kinetic energy [1–8]. On the basis of the developed mathematical model, a numerical simulation was carried out of both degenerate Fermi systems and a charge-symmetric Boltzmann plasma consisting of scalar charged particles and antiparticles [9–13].[1] These investigations revealed unique properties of cosmological models which are based on statistical systems of scalar charged particles with a phantom scalar interaction. However, since general results were obtained using numerical simulation methods, they can hardly be used for a description of asymptotic properties of the corresponding models. A large number of papers dedicated to qualitative studies of cosmological models (see, e.g., [16–18]). Using a combination of methods of the qualitative theory of ordinary differential equations and their numerical integration, in [19, 20], the asymptotic properties of a standard cosmological model based on a classical massive scalar field were investigated. Particularly, in these papers it has been shown that the set of Einstein-Klein–Gordon equations for a homogenous spatially flat cosmological model has a single singular point corresponding to zero values of the scalar field potential and its derivative, and this singular point can also be an attractive center or an attractive focus, or a saddle. Besides, a microscopic oscillating character of the invariant cosmological acceleration was revealed at approaching the singular point, with an average value corresponding to a nonrelativistic equation of state. In this paper we carry out a similar investigation for the "standard" cosmological model based on phantom fields. In this model, as opposed to those considered in [1–13], we do not take into account a contribution of ordinary matter, i.e., we consider free phantom fields without source.

2. BASIC RELATIONS FOR COSMOLOGY WITH A PHANTOM SCALAR FIELD

2.1. Equations of a Scalar Field with Self-Interaction

The Lagrange function of a phantom scalar field with mass m and self-interaction has the form [3]

$$L = -\frac{1}{8\pi}\left(g^{ik}\Phi_{,i}\Phi_{,k} + m^2\Phi^2 + \frac{\alpha}{2}\Phi^4\right), \quad (1)$$

where α is a coupling constant. The energy-momentum tensor relative to this function

$$T^{ik} = \frac{1}{8\pi}\Bigg(-2\Phi^{,i}\Phi^{,k} + g^{ik}\Phi_{,j}\Phi^{,j}v + g^{ik}m^2\Phi^2 + g^{ik}\frac{\alpha}{2}\Phi^4\Bigg) \quad (2)$$

has a negative kinetic term. If the covariant divergence of this tensor is zero, we have the following equation of a free phantom scalar field:

$$\Box\Phi - m_*^2\Phi = 0, \quad (3)$$

where

$$m_*^2 = m^2 + \alpha\Phi^2 \quad (4)$$

is the effective mass of a scalar boson. Equation (3) is different from the Klein–Gordon equation due to

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[1] Note that phantom scalar field have been extensively used for the description of wormholes and so-called black universes, see, e.g., [14, 15].